STEADY-STATE HEAT TRANSFER BETWEEN FLUIDS SEPARATED BY A THIN PARTITION

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The solution is obtained of the conjugate problem of steady-state convective heat exchange between two viscous fluids separated by a thin-walled wedge. The solution is employed in the design of heat exchangers of the "star" type.

1. Steady-state heat exchange is considered between two different viscous fluids separated by a thin-walled wedge of finite size with the flare angle equal to $\pi\beta$ (Fig. 1). The subscript α refers to quantities in the region α (α = 1 or 2). Hydrodynamic computations for the boundary layer employ the solution of the flow past the infinite wedge. It was shown in [1] that in this case the flux rate at the boundary of the boundary layer is given by

$$U_\alpha(x) = U_{0,\alpha}(x/L)^{m_\alpha},\ m_1 = \beta/(2-\beta),\ m_2 = (2-\beta)/\beta.$$

To solve our problem the equations of a stationary boundary layer are used, the heat-conduction equation for the wedge walls and the appropriate boundary conditions, the heat flux $q_0(y)$ for x = L being given in advance. In view of the symmetry of our model it suffices to solve the problem for one wedge wall only under the condition that

$$k_s \int_{-H}^{0} \frac{\partial T_s}{\partial x} dy \to 0,\quad x \to 0.$$

By introducing the dimensionless coordinates $x, y \to z = x/L$, $\eta_\alpha = \left[\frac{1}{2}(m_\alpha + 1)\mathrm{Re}_\alpha\right]^{1/2} z^{(m_\alpha - 1)/2}(-1)^{\alpha+1}(y + H_{2,\alpha})/L$ in the region α, $x, y \to z = x/L$, $\zeta = y/L$ on the wedge wall and introducing the flow function $\psi_\alpha = \nu_\alpha\{(2/(m_\alpha + 1))\mathrm{Re}_\alpha z^{m_\alpha+1}\}^{1/2} f_\alpha(\eta_\alpha)$, $\mathrm{Re}_\alpha = U_{0,\alpha}L/\nu_\alpha$ one can write the conjugate heat-exchange problem as follows:

$$f_\alpha''' + f_\alpha f_\alpha'' + \beta_\alpha[1-(f_\alpha')^2] = 0,\ f_\alpha(0) = f_\alpha'(0) = 0,\ f_\alpha'(\infty) = 1,\ \beta_\alpha = \begin{cases}\beta, & \alpha=1\\ 2-\beta, & \alpha=2\end{cases}, \tag{1}$$

$$\frac{\partial^2\Theta_\alpha}{\partial\eta_\alpha^2} + f_\alpha\mathrm{Pr}_\alpha\frac{\partial\Theta_\alpha}{\partial\eta_\alpha} = \frac{2\mathrm{Pr}_\alpha}{m_\alpha + 1} z f_\alpha' \frac{\partial\Theta_\alpha}{\partial z} - \mathrm{Pr}_\alpha\mathrm{Ec}_\alpha z^{2m_\alpha}(f_\alpha'')^2, \tag{2}$$

$$\frac{\partial^2\Theta_s}{\partial z^2} + \frac{\partial^2\Theta_s}{\partial\zeta^2} = 0, \tag{3}$$

$$(-1)^{\alpha+1} M_\alpha h z^{\frac{m_\alpha - 1}{2}} \left(\frac{\partial\Theta_\alpha}{\partial\eta_\alpha}\right)_{\eta_\alpha=0} = \left(\frac{\partial\Theta_s}{\partial\zeta}\right)_{\zeta=-h\delta_{2,\alpha}}, \tag{4}$$

$$\Theta_\alpha|_{\eta_\alpha=0} + \Theta_0\delta_{2,\alpha} = \Theta_s|_{\zeta=-h\delta_{2,\alpha}}, \tag{4), (5}$$

$$\Theta_\alpha(z, \infty) = 0,\quad \left(\frac{\partial\Theta_s}{\partial z}\right)_{z=1} = \mathrm{Nu}_0(\zeta),\quad \frac{\partial}{\partial z}\left(\int_{-h}^{0}\Theta_s d\zeta\right) \to 0 \text{ for } z \to 0; \tag{6)–(8}$$

$$\mathrm{Pr}_\alpha = \nu_\alpha\rho_\alpha c_\alpha/k_\alpha,\ \mathrm{Ec}_\alpha = U_{0,\alpha}^2/c_\alpha T_{\infty,1},$$

$$M_\alpha = (k_\alpha/k_s)\left[\frac{1}{2}(m_\alpha + 1)\mathrm{Re}_\alpha\right]^{1/2}(L/H),\ h = H/L,$$

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$$\mathrm{Nu}_0 = -Lq_0/k_sT_{\infty,1},\ T_s = T_{\infty,1}(\Theta_s + 1),\ T_\alpha = T_{\infty,1}(\Theta_\alpha + \Theta_0\delta_{2,\alpha} + 1),$$
$$\Theta_0 = (T_{\infty,2}/T_{\infty,1}) - 1.$$

Similarly as in [2], Eq. (3) and the conditions (7) and (8) are averaged over the dimensionless thickness h of the wall of the wedge. Assuming that the wedge walls are adequately thin one can set

$$\Theta_s|_{\zeta=0} = \frac{1}{h}\int_{-h}^{0}\Theta_s d\xi = \Theta_s|_{\zeta=-h}. \tag{9}$$

Thus by using the conditions (4), (5), and (9) one obtains

$$\frac{\partial^2\Theta_1|_{\eta_1=0}}{\partial z^2} + M_1 z^{\frac{m_1-1}{2}}\left(\frac{\partial\Theta_1}{\partial\eta_1}\right)_{\eta_1=0} + M_2 z^{\frac{m_2-1}{2}}\left(\frac{\partial\Theta_2}{\partial\eta_2}\right)_{\eta_2=0} = 0, \tag{10}$$

$$\left(\frac{\partial\Theta_1|_{\eta_1=0}}{\partial z}\right)_{z=1} = \bar{\mathrm{Nu}}_0,\quad \bar{\mathrm{Nu}}_0 = \frac{1}{h}\int_{-h}^{0}\mathrm{Nu}_0(\zeta)\,d\zeta, \tag{11}$$

$$\left(\frac{\partial\Theta_1|_{\eta_1=0}}{\partial z}\right)_{z=0} = 0,\quad \Theta_1|_{\eta_1=0} = \Theta_2|_{\eta_2=0} + \Theta_0. \tag{12),(13}$$

The solution of the heat problem (2), (6), (10)-(13) is sought in the form (which is also suitable for any m_1, m_2),

$$\Theta_\alpha(z,\ \eta_\alpha) = \Theta_\alpha^{(1)}(z,\ \eta_\alpha) + \Theta_\alpha^{(2)}(z,\ \eta_\alpha) + \Theta_\alpha^{(3)}(z,\ \eta_\alpha), \tag{14}$$

$$\Theta_\alpha^{(i)} = \sum_{k,n\geq 0}^{+\infty} Y_{\alpha,k,n}^{(i)}(\eta_\alpha)\exp\left\{\frac{1}{2}\mathrm{Pr}_\alpha\left(F_{\alpha,k,n}^{(i)} - F_\alpha\right)\right\} z^{\rho^{(i)}+\mu_{k,n}^{(i)}},\ Y_{\alpha,0,0}^{(i)}(0) = 0,\ (i = 1,\ 2), \tag{14'}$$

$$F_{\alpha,k,n}^{(i)} = \int_0^{\eta_\alpha} f_{\alpha,k,n}^{(i)} d\eta_\alpha,\ F_\alpha = \int_0^{\eta_\alpha} f_\alpha d\eta_\alpha,\ f_{\alpha,k,n}^{(i)\prime} + \frac{1}{2}\mathrm{Pr}_\alpha f_{\alpha,k,n}^{(i)2} = \lambda_{\alpha,k,n}^{(i)} f_\alpha' + \frac{1}{2}\mathrm{Pr}_\alpha f_\alpha^2,\ f_{\alpha,k,n}^{(i)}(0) = 0,$$

$$\lambda_{\alpha,k,n}^{(i)} = 1 + \frac{4\mu_{k,n}^{(i)}}{m_\alpha + 1},\ \mu_{k,n}^{(i)} = \rho^{(i)} + \frac{m_1+3}{2}k + \frac{m_2+3}{2}n, \tag{15}$$

$$\rho^{(i)} = \begin{cases} 2m_1, & i = 1,\\ 2m_2, & i = 2,\\ 0, & i = 3.\end{cases}$$

By inserting (14) in (2), (10), (13) and comparing the coefficients of equal powers of z for each $\Theta_\alpha^{(i)}$ the following recurrence relations are obtained

$$Y_{\alpha,k,n}^{(i)}(\eta_\alpha) = -Y_{\alpha,k,n}^{(i)\prime}(0)\int_{\eta_\alpha}^{+\infty}\exp\{-\mathrm{Pr}_\alpha F_{\alpha,k,n}^{(i)}\}\,d\eta_\alpha + \delta_{\alpha,i}\delta_{0,k}\delta_{0,n}\Phi_\alpha(\eta_\alpha),$$

$$\Phi_\alpha = \mathrm{Pr}_\alpha\mathrm{Ec}_\alpha\int_{\eta_\alpha}^{+\infty}\left\{\int_0^{\eta_\alpha}[f_\alpha''(\eta_\alpha')]^2\exp\left\{\frac{1}{2}\mathrm{Pr}_\alpha(F_\alpha(\eta_\alpha') + F_{\alpha,0,0}^{(\alpha)}(\eta_\alpha'))\right\}\right.$$
$$\left.\times\, d\eta_\alpha'\right\}\exp\{-\mathrm{Pr}_\alpha F_{\alpha,0,0}^{(\alpha)}\}\,d\eta_\alpha,^{*}\ Y_s^{(i)} = T_{s,s-1}^{(i)}\times Y_{s-1}^{(i)} - R_s^{(i)},\ s\geq 1. \tag{16}$$

In an expanded form (16) is as follows:

$$\begin{bmatrix} Y_{1,0,s}^{(i)}(0)\\ Y_{1,1,s-1}^{(i)}(0)\\ \cdots\\ Y_{1,s,0}^{(i)}(0)\end{bmatrix} = \begin{bmatrix} t_{0,s}^{(i)} & 0 & \ldots & 0\\ t_{1,s-1}^{(i)} & t_{1,s-1}^{(i)} & \ldots & 0\\ \cdots & & & \\ 0 & 0 & \ldots & t_{s,0}^{(i)}\end{bmatrix}\times\begin{bmatrix} Y_{1,0,s-1}^{(i)}(0)\\ Y_{1,1,s-2}^{(i)}(0)\\ \cdots\\ Y_{1,s-1,0}^{(i)}(0)\end{bmatrix} - \begin{bmatrix} R_{0,s}^{(i)}\\ R_{1,s-1}^{(i)}\\ \cdots\\ R_{s,0}^{(i)}\end{bmatrix},$$

$$t_{k,n}^{(i)} = M_2/\psi_{k,n}^{(i)} I_{2,k,n-1}^{(i)},\quad t_{k,n}^{(i)} = M_1/\psi_{k,n}^{(i)} I_{1,k-1,n}^{(i)},$$

$$I_{\alpha,k,n}^{(i)} = \int_0^{+\infty}\exp\{-\mathrm{Pr}_\alpha F_{\alpha,k,n}^{(i)}\}\,d\eta_\alpha.^{*}$$

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* In Appendix 1 the formulas are given for evaluating approximately $I_{\alpha,k,n}^{(i)}$, $\Phi_\alpha(0)$.

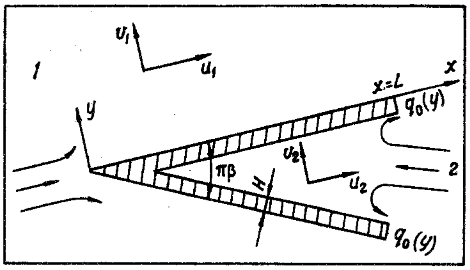


Fig. 1. Model of heat exchange between fluids separated by a thin-walled wedge.

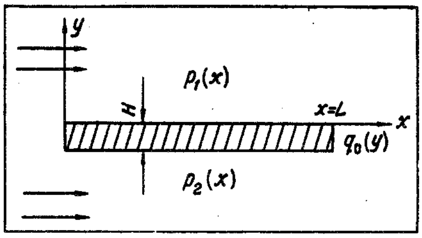


Fig. 2. Model of heat exchange between fluids separated by a thin plate. $(\partial T_s/\partial x)_{x=0}=0$; $P_\alpha(x) = P_{0,\alpha} - \rho_\alpha U_{0\alpha}^2/2(x/L)^{2m_\alpha}$, $m_\alpha \geq 0$.

$$\psi_{k,n}^{(i)} = \mu_{k,n}^{(i)}(\mu_{k,n}^{(i)}-1),\ R_{k,n}^{(i)} = l_{1,0}^{(i)}\Phi_1(0)\delta_{1,i}\delta_{0,k-1}\delta_{0,n} + l_{0,1}^{(i)}(\Phi_2(0)\delta_{2,i} + \Theta_0\delta_{3,i})\delta_{0,k}\delta_{0,n-1}.$$

The recurrence relations (16) (together with (14')) are solved for s = 1, 2 as follows:

$$Y_s^{(i)} = \delta_{i,3}\, T_{s,0}^{(3)} \times Y_0^{(3)} - T_{s,1}^{(i)} \times R_1^{(i)},$$
$$T_{s,\lambda}^{(i)} = \begin{cases} T_{s,s-1}^{(i)} \times \ldots T_{\lambda+1,\lambda}^{(i)}, & s > \lambda, \\ E, & s = \lambda. \end{cases} \tag{17}$$

To find $Y_0^{(3)} = Y_{1,0,0}^{(3)}(0) = \Theta_1(0, 0)$ one employs the boundary condition (11). (It should be mentioned that the boundary condition (12) is already satisfied in view of the assumed form of the solution (14), (14').) The insertion of (14) into (11) (using (17)) results in

$$Y_0^{(3)} = \frac{\sum_{i=1}^{3}\left[\sum_{s=1}^{+\infty} L_s^{(i)} \times T_{s,1}^{(i)}\right] \times R_1^{(i)} + \tilde{\mathrm{N}}\mathrm{u}_0}{\sum_{s=1}^{+\infty} L_s^{(3)} \times T_{s,0}^{(3)}},\quad L_s^{(i)} = \frac{d}{dz}(z^{\rho^{(i)}} Z_s)_{z=1},$$

$$Z_s = [\underbrace{\ldots\ z^{\frac{m_1+3}{2}k + \frac{m_2+3}{2}(s-k)}}_{s+1}]. \tag{17'}$$

The temperatures of the wedge wall and of the heat flux from the fluid to the wedge are now written down,

$$T_s = T_{\infty,1}\left\{1 + Y_0^{(3)}\left(1 + \sum_{s=1}^{+\infty} Z_s \times T_{s,0}^{(3)}\right) - \sum_{i=1}^{3} z^{\rho^{(i)}}\left[\sum_{s=1}^{+\infty} Z_s \times T_{s,1}^{(i)}\right] \times R_1^{(i)}\right\}, \tag{18}$$

$$q_s|_{y=-H\delta_{2,\alpha}} = k_s(T_{\infty,1}/L)\, M_\alpha h z^{\frac{m_\alpha-1}{2}}\left\{(\Theta_0/I_{2,0,0}^{(3)})\,\delta_{2,\alpha} + z^{2m_\alpha}(\Phi_\alpha(0)/I_{\alpha,0,0}^{(\alpha)})\right.$$

$$\left. Y_0^{(3)}\left[(1/I_{\alpha,0,0}^{(3)}) + \sum_{s=1}^{+\infty} Z_s \times B_{s,\alpha}^{(3)} \times T_{s,0}^{(3)}\right] + \sum_{i=1}^{3} z^{\rho^{(i)}}\left[\sum_{s=1}^{+\infty} Z_s \times B_{s,\alpha}^{(i)} \times T_{s,1}^{(i)}\right] \times R_1^{(i)}\right\}, \tag{19}$$

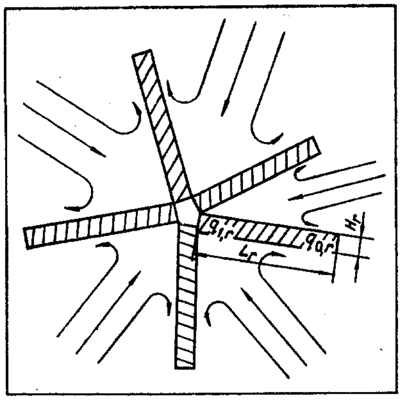
$$\begin{bmatrix} 1/I_{\alpha,0,s}^{(i)} & 0 & \ldots & 0 \\ 0 & 1/I_{\alpha,1,s-1}^{(i)} & \ldots & 0 \\ 0 & 0 & \ldots & 1/I_{\alpha,s,0}^{(i)} \end{bmatrix} = B_{s,\alpha}^{(i)}.$$

From (18) and (19) one can find $\mathrm{Nu}_\alpha = -(Lq_s|_{y=-H\delta_{2,\alpha}})/k_\alpha(T_s - T_{\infty,\alpha})$. It is not difficult to show with the aid of the inequalities (II.4)-(II. 6) (Appendix II) that the series appearing in the expressions (18)-(19) are convergent.

It is noted that the solution (18), (19) of the conjugate problem (1)-(8) was obtained by neglecting the interaction between the boundary layers on different walls of the same region α as well as the effect of the ends of the edge on the flow.

2. The above solution is suitable for describing the heat exchange for the model (Fig. 2). (For any m_1, m_2). In this case to calculate β_α appearing in (1) the formula $\beta_\alpha = 2m_\alpha/(m_\alpha + 1)$ should be used. The expressions (18)-(19) simplify considerably for $m_1 = m_2 = 0$, $\mathrm{Pr}_1 = \mathrm{Pr}_2$:

$$Y_0^{(3)} = (M_1\Phi_1(0) + M_2(\Phi_2(0) + \Theta_0))/(M_1 + M_2) + \tilde{\mathrm{N}}\mathrm{u}_0 \Big/ \left(\frac{dD_1^{(3)}}{dz}\right)_{z=1},$$



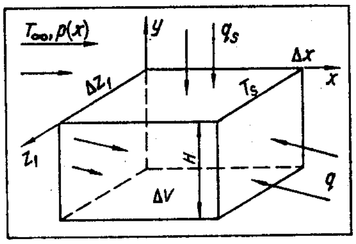


Fig. 3 Fig. 4

Fig. 3. Heat exchange for a "star" type construction. In region 1 a source is active with output E(r = 1, 2, . . . , N).

Fig. 4. A model showing the physical meaning of the parameter M. $p(x) = p_0 - \rho U_0^2/2(x/L)^{2m}$, $m \geq 0$.

$$T_s = T_{\infty,1}\left[1 + Y_0^{(3)} \mp \tilde{\mathrm{N}}\mathrm{u}_0 D_1^{(3)}(\sigma) \Big/ \left(\frac{\mathrm{d}D_1^{(3)}}{\mathrm{d}z}\right)_{z=1}\right], \quad \sigma = (M_1 + M_2)\, z^{3/2},$$

$$q_s\big|_{y=0} = k_s (T_{\infty,1}/L)\, M_1 \left(h/z^{1/2}\right)$$

$$\times \left\{M_2(\Phi_1(0) - \Phi_2(0) - \Theta_0)/(M_1 + M_2)\, I_{1,0,0}^{(3)} - \tilde{\mathrm{N}}\mathrm{u}_0 \Lambda_1^{(3)}(\sigma) \Big/ \left(\frac{\mathrm{d}D_1^{(3)}}{\mathrm{d}z}\right)_{z=1}\right\},$$

$$q_s\big|_{y=-H} = k_s (T_{\infty,1}/L)\, M_2 \left(h/z^{1/2}\right)$$

$$\times \left\{M_1(\Theta_0 + \Phi_2(0) - \Phi_1(0))/(M_1 + M_2)\, I_{1,0,0}^{(3)} - \tilde{\mathrm{N}}\mathrm{u}_0 \Lambda_1^{(3)}(\sigma) \Big/ \left(\frac{\mathrm{d}D_1^{(3)}}{\mathrm{d}z}\right)_{z=1}\right\}.$$

In contrast to [3], in the solution (18)-(19) obtained by us for $m_1 = m_2 = \mathrm{Nu}_0 = 0$ the dissipative terms in the energy equations for fluids and the lengthwise heat conduction of the plate are taken into account.

3. Employing the same method of solution one can also evaluate the heat exchange of the model shown in Fig. 2 but with the boundary condition

$$k_s \left(\frac{\mathrm{d}T_s}{\mathrm{d}x}\right)_{x=0} = -q_1(y). \tag{20}$$

By carrying out the previously described transformations, the boundary condition

$$\left(\frac{\mathrm{d}\Theta_1|_{\eta_1=0}}{\mathrm{d}z}\right)_{z=0} = \tilde{\mathrm{N}}\mathrm{u}_1, \quad \tilde{\mathrm{N}}\mathrm{u}_1 = -\frac{L}{k_s T_{\infty,1} h} \int\limits_{-h}^{0} q_1 \mathrm{d}\zeta, \tag{21}$$

is obtained which should now be used instead of (12). For (21) to be taken into account we introduce in (14) the additional term $\Theta_\alpha^{(4)}(z, \eta_\alpha)$ which can be determined by the formula (14') with $\rho^{(4)} = 1$. This results in additional terms for (18),

$$\tilde{\mathrm{N}}\mathrm{u}_1 z \left(1 + \sum_{s=1}^{+\infty} Z_s \times T_{s,0}^{(4)}\right) \tag{22}$$

and in the square brackets of (17') and (19) one now has respectively

$$-\tilde{\mathrm{N}}\mathrm{u}_1\left(1 + \sum_{s=1}^{+\infty} L_s^{(4)} \times T_{s,0}^{(4)}\right), \quad -\tilde{\mathrm{N}}\mathrm{u}_1 z\left[(1/I_{\alpha,0,0}^{(4)}) + \sum_{s=1}^{+\infty} Z_s \times B_{s,\alpha}^{(4)} \times T_{s,0}^{(4)}\right]. \tag{21'}$$

These solutions enable one to compute the heat exchange for a construction shown in Fig. 3. The expressions (17'), (18), (22), (22') for $T_{s,r}(0)$ can be represented by $T_{s,r}(0) = \varepsilon_r^{(1)} + \tilde{q}_{1,r}/\varepsilon_r^{(2)}$ ($\varepsilon_r^{(j)}$ are introduced to shorten the notation).

The solution of the system of (N + 1) equations obtained from the heat-balance condition in the region 1 is as follows:

$$\tilde{T}_s = \left(E + \sum_{r=1}^{N} \varepsilon_r^{(1)} \varepsilon_r^{(2)} H_r\right) / \left(\sum_{r=1}^{N} \varepsilon_r^{(2)} H_r\right), \quad \tilde{q}_{1,r} = \varepsilon_r^{(2)} (\tilde{T}_s - \varepsilon_r^{(1)}),$$

where $\tilde{T}_S$ is the temperature of the region 1.

By inserting $\tilde{T}_S$, $\tilde{q}_{1,r}$ in (18), (19), (22), (22') thermal fields are obtained in each plate.

4. In conclusion the physical meaning of the parameter M will be discussed in more detail.

During the time Δt there comes from the fluid into the volume ΔV the amount of heat $q_S = k(\partial T/\partial y)_{y=0}\Delta x \Delta z_1 \Delta t \sim k[(T_S - T_\infty)/\delta]\Delta x \Delta z_1 \Delta t$, where $\delta \sim L[2/(m+1)\mathrm{RePr}]^{1/2}$ is the thickness of the heat boundary layer. During the same period of time there comes into the same volume some amount of heat propagated along the body $q = k_S(\partial^2 T_S/\partial x^2)\Delta x \Delta z_1 H \Delta t \sim k_S[(T_S - T_\infty)/L_2]\Delta x \Delta z_1 H \Delta t$. Hence one can see that

$$q_s/q \sim M\sqrt{\mathrm{Pr}}.$$

Thus the similitude criterion which involves the heat parameters of the fluid as well as of the body shows how many times the heat flux from the fluid is greater than the heat flux propagated in the body. It should also be mentioned that such similitude criterion is especially characteristic for conjugate heat-exchange problems.

APPENDIX I

a) The function $I(\mathrm{Pr}, \lambda) = \int_0^{+\infty} \exp\{-\mathrm{Pr}\,Q\}d\eta$ is studied where

$$Q = \int_0^{\eta} \varphi\, d\eta, \quad \frac{\partial \varphi}{\partial \eta} + \frac{\mathrm{Pr}\varphi^2}{2} = \lambda f' + \frac{\mathrm{Pr} f^2}{2}, \quad \varphi|_{\eta=0} = 0. \tag{I.1}$$

From (I.1) one easily obtains

$$\left\{\begin{aligned} \frac{\partial I}{\partial \mathrm{Pr}} &= -\int_0^{+\infty} \partial\eta \int_0^{\eta} d\eta' \int_0^{\eta'} [\lambda f'(\eta'') + \mathrm{Pr} f^2(\eta'')]\exp\{\mathrm{Pr}[Q(\eta'') - Q(\eta') - Q(\eta)]\}d\eta'', \\ \frac{\partial I}{\partial \lambda} &= -\mathrm{Pr}\int_0^{+\infty} d\eta \int_0^{\eta} d\eta' \int_0^{\eta'} f'(\eta'')\exp\{\mathrm{Pr}[Q(\eta'') - Q(\eta') - Q(\eta)]\}d\eta''. \end{aligned}\right. \tag{I.2}$$

It was shown in [1] that $f' \geq 0$ for $\beta \geq 0$; therefore, (I.2) implies that

$$\frac{\partial I}{\partial \mathrm{Pr}} < 0, \quad \frac{\partial I}{\partial \lambda} < 0. \tag{I.3}$$

b) By using the method of steepest descent one can derive the following approximate formulas [2]:

$I^{(i)}_{\alpha,k,n} = I(\mathrm{Pr} = \mathrm{Pr}_\alpha,\ \lambda = \lambda^{(i)}_{\alpha,k,n},\ \beta = \beta_\alpha,\ a = f''_\alpha(0))$, $\Phi_\alpha(0) = \Phi(\mathrm{Pr} = \mathrm{Pr}_\alpha,\ \lambda = \lambda^{(\alpha)}_{\alpha,0,0},\ \beta = \beta_\alpha,\ a = f''_\alpha(0))$,

$$I \cong \sum_{r=0}^{+\infty} d_r \Gamma\left(\frac{r+1}{3}\right)\mathrm{Pr}^{-\frac{r+1}{3}}, \quad d_0 = \frac{1}{3}\left(\frac{6}{\lambda a}\right)^{1/3}, \quad d_1 = \frac{\beta}{18a}$$

$$\times\left(\frac{6}{\lambda a}\right)^{2/3}, \quad d_2 = \frac{\beta^2}{8\lambda a^3}, \quad d_3 = \left(\frac{6}{\lambda a}\right)^{1/3}\left[\frac{1-2\beta}{45\lambda} + \frac{\mathrm{Pr}(\lambda^2-1)}{15\lambda^2}\right.$$

$$\left.+\frac{35\beta^3}{648\lambda a^4}\right], \quad d_4 = \left(\frac{6}{\lambda a}\right)^{2/3}\left[\frac{\beta(1-5\beta)}{378a\lambda} + \frac{\mathrm{Pr}(\lambda^2-1)\beta}{63\lambda^2 a} + \frac{385\beta^4}{15552\lambda a^5}\right],$$

$$d_5 = \frac{3}{\lambda^2 a^2}\left[\frac{\beta^2(1-12\beta)}{840a} + \frac{13\mathrm{Pr}\,\beta^2(\lambda^2-1)}{840\lambda a} + \frac{3\beta^5}{128a^5}\right], \ldots$$

To evaluate $I^{(i)}_{\alpha,k,n}$ in the case of slow convergence of the series the Euler transformation should be used.

$$\Phi \cong \mathrm{EcPr}^{1/3}\sum_{r=0}^{6} t_r \mathrm{Pr}^{-r/3}, \quad d_0^* = \frac{a^2}{3}\left(\frac{6}{\lambda a}\right)^{1/3}, \quad d_1^* = -\frac{11\beta a}{18}\left(\frac{6}{\lambda a}\right)^{2/3}, \quad d_2^* = \frac{9\beta^2}{8\lambda a},$$

$$d_3^* = \frac{2}{\lambda a}\left(\frac{6}{\lambda a}\right)^{1/3}\left\{\frac{215\beta^3}{1296a} + \frac{a^3}{15}\left[\frac{1+178\beta}{6} - \frac{\Pr\beta(1+12\beta)}{1+4\beta}\right]\right\},\ A_r^* = 3d_r,$$

$$t_r = \begin{cases} \sum_{k=0}^{r} d_k^* A_{r-k}^* \omega_{k,r}, & r \leqslant 3, \\ \sum_{k=r-3}^{3} d_k^* A_{r-k}^* \omega_{k,r}, & r > 3, \end{cases}$$

$$\omega_{k,r} = \begin{cases} \sum_{n=0}^{N-1} X_{k,r}(n) + \frac{1}{2} X_{k,r}(N) + J_{1,r} + J_{2,k,r}, & \Pr > 2, \\ \sum_{n=0}^{N} X_{k,r}(n), & 1 < \Pr \leqslant 2, \end{cases}$$

$$J_{1,r} = \begin{cases} \frac{1}{r-1}\left[-N^{\frac{r-1}{3}} \exp(-\varepsilon N) + \varepsilon^{\frac{1-r}{3}} \Gamma\left(\frac{r+2}{3},\ \varepsilon N\right)\right], & r \neq 1, \\ -\frac{1}{3} \mathrm{Ei}(-\varepsilon N), & r = 1, \end{cases}$$

$$J_{2,k,r} = \begin{cases} \frac{\varkappa_{k,r}}{r-4}\left[-N^{\frac{r-4}{3}} \exp(-\varepsilon N) + \varepsilon^{\frac{4-r}{3}} J_{1,r}\right] & r \neq 4, \\ -\frac{\varkappa_{k,r}}{3} \mathrm{Ei}(-\varepsilon N), & r = 4, \end{cases}$$

$$\varkappa_{k,r} = \frac{r^2 + r - 6k - 8}{18},\ X_{k,r}(n) =$$

$$= \frac{\Gamma\left(n + \frac{r+2}{3}\right)}{\Gamma(n+1)(3n+k+1)}\left(\frac{\Pr - 2}{\Pr}\right)^n,\ \varepsilon = -\ln\left(1 - \frac{2}{\Pr}\right),$$

$\Gamma(n, \tau)$ denotes the incomplete gamma-function; $\mathrm{Ei}(\tau)$ denotes the integral exponential function: N is a suitably large number.

APPENDIX II

It can be seen from (15) that for $k + n = s$ one has

$$\mu_{1,s}^{(i)} \equiv \rho^{(i)} + \frac{s}{2}(p_1 + 3) \leqslant \mu_{k,n}^{(i)} \leqslant \mu_{2,s}^{(i)} \equiv \rho^{(i)} + \frac{s}{2}(p_2 + 3), \tag{II.1}$$

$$\lambda_{1,s}^{(i)} \equiv 1 + \frac{4\mu_{1,s}^{(i)}}{p_2 + 1} \leqslant \lambda_{\alpha,k,n}^{(i)} \leqslant \lambda_{2,s}^{(i)} \equiv 1 + \frac{4\mu_{2,s}^{(i)}}{p_1 + 1}, \tag{II.2}$$

where

$$p_1 = \min\{m_1, m_2\}, \qquad p_2 = \max\{m_1, m_2\}.$$

By using (I.3) and (II.2) one can easily prove the inequality

$$K_{1,s-1}^{(i)} = I(\Pr_{\mathrm{I}}, \lambda_{2,s-1}^{(i)}) \leqslant I_{2,k,n-1}^{(i)},\ I_{1,k-1,n}^{(i)} \leqslant K_{2,s-1}^{(i)} = I(\Pr_{\mathrm{II}}, \lambda_{1,s-1}^{(i)}), \tag{II.3}$$

where $\Pr_{\mathrm{I}} = \max\{\Pr_1, \Pr_2\}$, $\Pr_{\mathrm{II}} = \min\{\Pr_1, \Pr_2\}$.

Thus, the following estimates from (II.1) and (II.3) can be obtained by using the explicit form of the matrix $T_{s,s-1}^{(i)}$:

$$D_2^{(3)} \leqslant \sum_{s=1}^{+\infty} Z_s \times T_{s,0}^{(3)} \leqslant D_1^{(3)}, \quad D_2^{(i)} \tilde{R}_i \leqslant \sum_{s=1}^{+\infty} Z_s \times T_{s,1}^{(i)} \times R_1^{(i)} \leqslant D_1^{(i)} \tilde{R}_i, \tag{II.4}$$

$$\left(\frac{dD_2^{(3)}}{dz}\right)_{z=1} \leqslant \sum_{s=1}^{+\infty} L_s^{(3)} \times T_{s,0}^{(3)} \leqslant \left(\frac{dD_1^{(3)}}{dz}\right)_{z=1},$$

$$\frac{d}{dz}\left(z^{\rho^{(i)}} D_2^{(i)} \tilde{R}_i\right)_{z=1} \leqslant \sum_{s=1}^{+\infty} L_s^{(i)} \times T_{s,1}^{(i)} \times R_1^{(i)} \leqslant \frac{d}{dz}\left(z^{\rho^{(i)}} D_1^{(i)} \tilde{R}_i\right)_{z=1} \tag{II.5}$$

$$\Lambda_2^{(3)} \leqslant \sum_{s=1}^{+\infty} Z_s \times B_{s,\alpha}^{(3)} \times T_{s,0}^{(3)} \leqslant \Lambda_1^{(3)}, \quad \Lambda_2^{(i)} \tilde{R}_i \leqslant \sum_{s=1}^{+\infty} Z_s \times B_{s,\alpha}^{(i)} \times T_{s,1}^{(i)} \times R_1^{(i)} \leqslant \Lambda_1^{(i)} \tilde{R}_i, \tag{II.6}$$

$$\tilde{R}_i = \Phi_1(0)\left[1 + z^{\frac{m_2-m_1}{2}} (M_2/M_1)\right]^{-1} \delta_{1,i} + (\Phi_2(0)\,\delta_{2,i} + \Theta_0 \delta_{3,i})\left[1 + z^{\frac{m_1-m_2}{2}} (M_1/M_2)\right]^{-1},$$

$$\gamma_{j,s}^{(i)} = \frac{(2/(p_j+3))^{2s}\,\Gamma(1+2\rho^{(i)}/(p_j+3))\,\Gamma(1+2(\rho^{(i)}-1)/(p_j+3))}{\Gamma(s+1+2\rho^{(i)}/(p_j+3))\,\Gamma(s+1+2(\rho^{(i)}-1)/(p_j+3))\prod_{r=0}^{s-1} K_{j,r}^{(i)}},$$

$$\sigma = M_1 z^{\frac{m_1+3}{2}} + M_2 z^{\frac{m_2+3}{2}},$$

$$D_j^{(i)} = \sum_{s=1}^{+\infty} \gamma_{j,s}^{(i)} \sigma^s, \qquad \Lambda_j^{(i)} = \sum_{s=1}^{+\infty} \frac{\gamma_{j,s}^{(i)}}{K_{j,s}^{(i)}} \sigma^s.$$

NOTATION

$T_\infty (T, T_S)$	is the temperature of incident fluid (of boundary layer, wedge);
$k(k_S)$	is the heat conduction coefficient of fluid (wedge);
ν	is the kinematic viscosity;
c	is the fluid specific heat capacity;
ρ	is the fluid density.

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